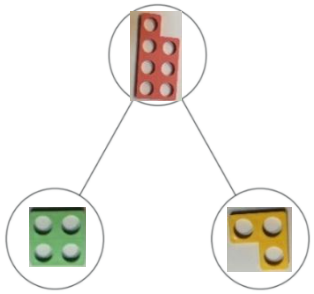
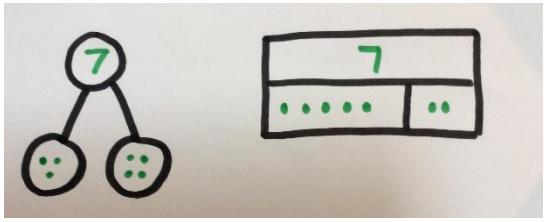
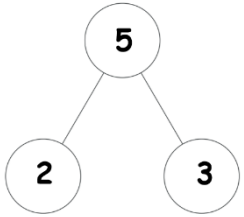
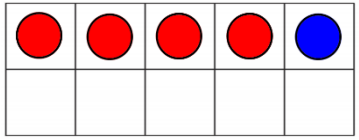
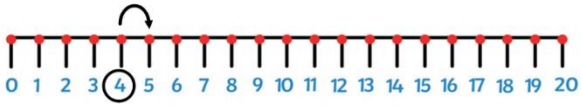
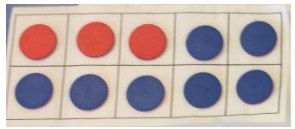
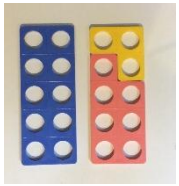
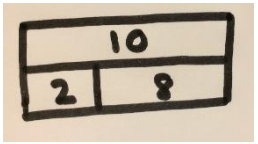
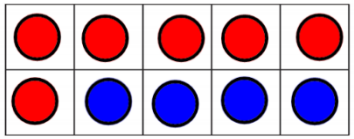
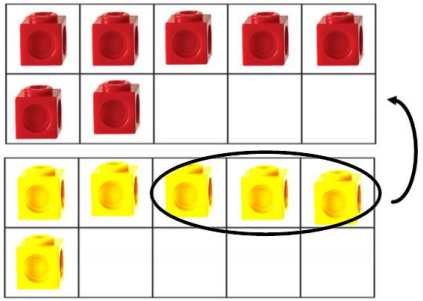
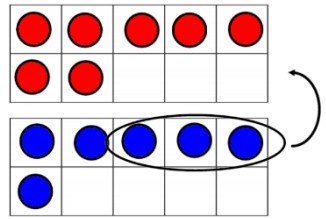
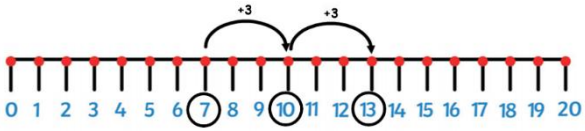
Calculation Policy

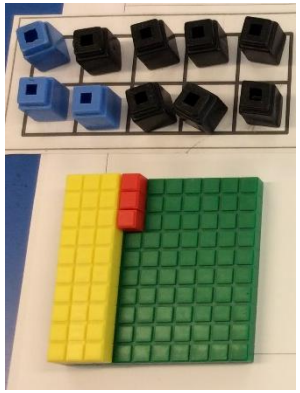
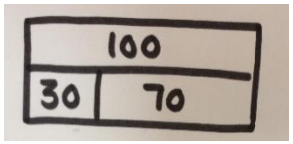
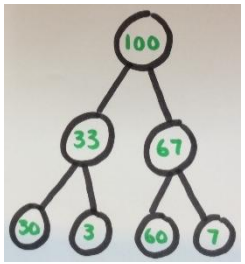
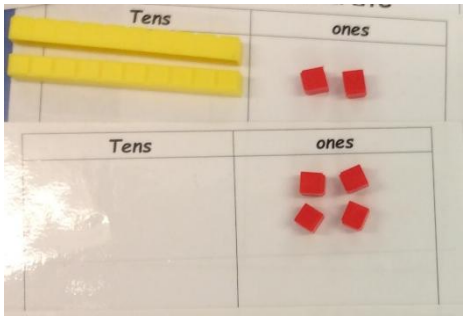
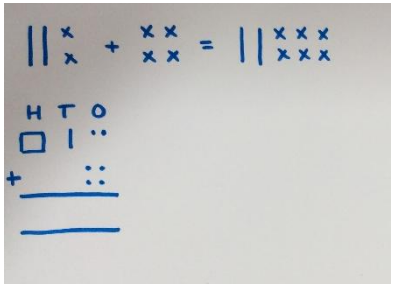
This policy was developed by the staff at Heage Primary School following the White Rose Planning Scheme. It supports the development of conceptual understanding using a concrete, pictorial, abstract approach covering the following operations:

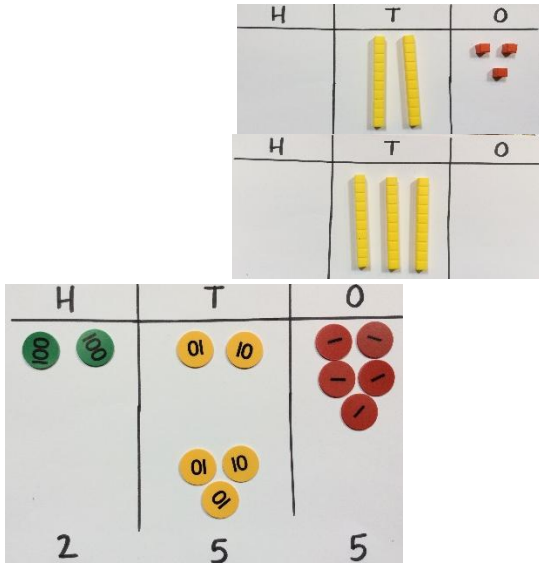
- Addition
- Subtraction
- Multiplication
- Division

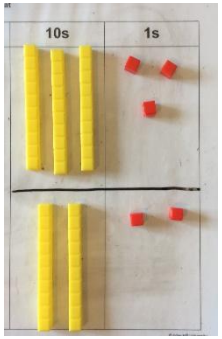
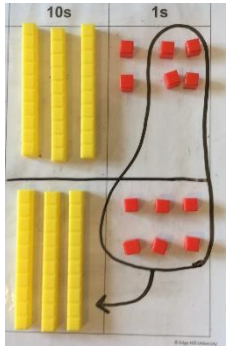
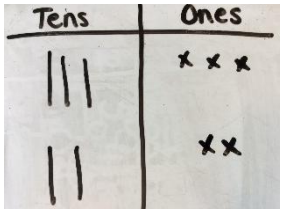
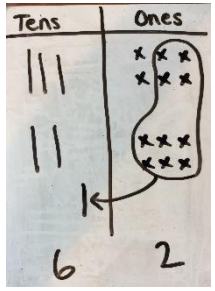
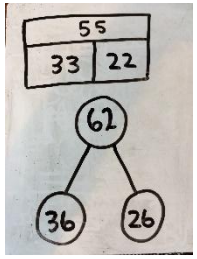
Progression in Addition

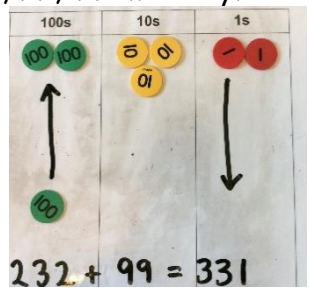
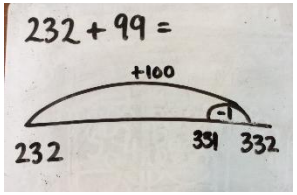
	CONCRETE	PICTORIAL	ABSTRACT
Combining two parts to make a whole.	Children will be introduced to the part-whole model and use this to combine two parts.  	Children will be introduced to the part-whole model and use this to combine two parts. Bar models will be introduced early to accompany the concept of a part-whole relationship. 	Children complete part-whole models writing in the digits and then beginning to record calculations using symbols (+ and =). ***Note: Children need to have an understanding that the (=) sign shows equality rather than just an answer.***  **2 + 3 = 5** **5 = 2 + 3**
Counting on.	Children use a variety of manipulatives to make the number and then add more. Children are encouraged to recognise that calculations can be done in any order and encouraged to represent the larger number first. 	Children represent on five and ten frames or use a number line to count on.  	**"1 more than 4 is 5"** **4 + 1 = 5**

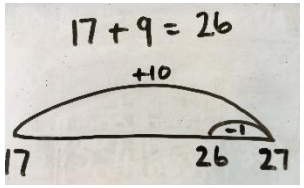
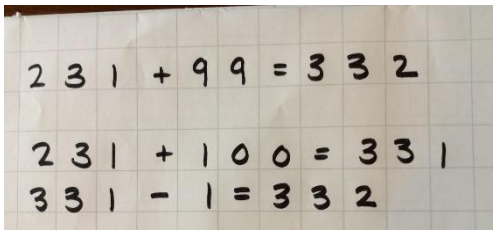
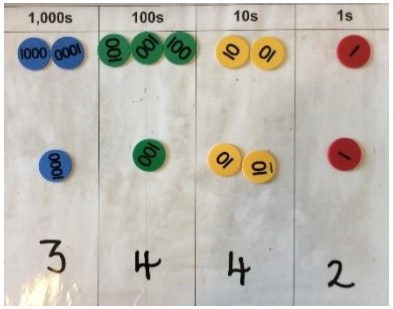
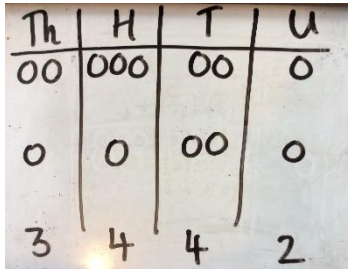
Number bonds to ten.	Children explore number bonds (first to 5, then to ten) using a variety of manipulatives including ten frames and Numicon.  	Children will represent using part-whole models and ten frames.  	Children will begin to recognise that calculations can be done in any order. **"3 reds and 7 blues make 10"** **"7 blues and 3 reds make 10"** **3 + 7 = 10** **7 + 3 = 10**
Re-grouping to make ten	Children use two ten frames and two colour counters or cubes to make their numbers individually, then re-group to make ten. Lots of consolidation will be required for this step. 	Children draw out their own version of the ten frames or use a number line to represent the re-grouping.  	Children represent calculations in a number sentence. **7 + 6 = 13** **7 + 3 + 3 = 13**

Number bods to 100	Children use ten frames and Base 10 apparatus to explore number bonds to ten, first in multiples of ten, seeing the relationship between number bonds to ten. Then they advance to using 2-digit numbers to find the complement to 100. 	Children use part-whole models and bar models to represent number bonds to 100, exploring the relationship between the tens and ones. They also represent their thinking using "counting on" with a number line.   	Children are confident in representing calculations using an expression, where they understanding the meaning of the (=) sign. **3 + 7 = 100** **30 + 70 = 100** **100 = 30 + 70**
Adding 2d or 3d to a 1d number	Children use a variety of manipulatives and place value grids to represent the larger number before adding the "ones". 	Children use drawing to represent the calculations. ***They will need to recognise that if the value of the ones doesn't add to more than 10 then the rest of the digits stay the same.*** 	Children represent calculations abstractly and begin to represent using the columnar method. **12 + 4 = 16** **12** **+ 4**

Adding a 2d or 3d number and tens	Children use a variety of manipulatives and place value grids to represent the larger number before adding the "tens". As children move into Year 3 they will begin to represent numbers using place value counters instead of Base 10. 	Children use drawing to represent the calculations. ***They will need to recognise that if the value of the tens doesn't add to more than 10 (exchanging for a hundred) then the rest of the digits stay the same.*** 	Children represent calculations abstractly and begin to represent using the columnar method. **12 + 20 = 32** **20** **+12**

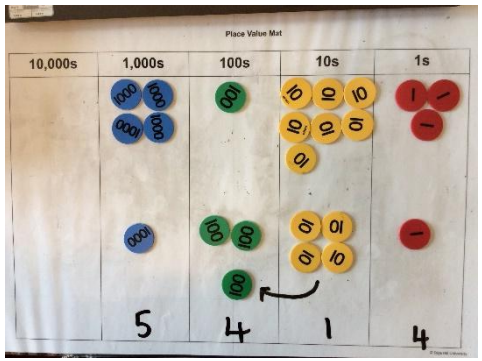
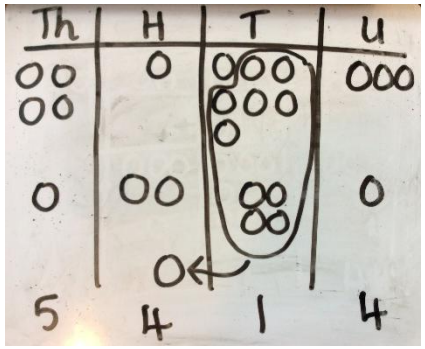
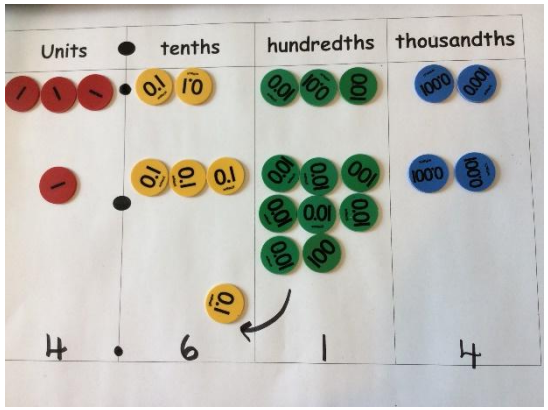
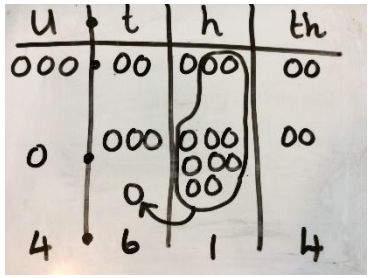
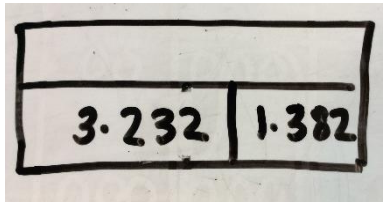
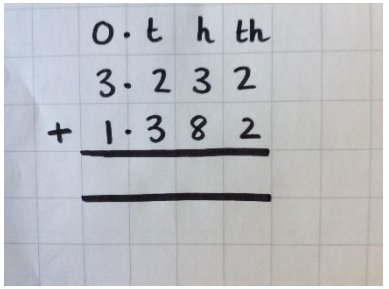
Adding two or more 2d or 3d numbers	Children learn to represent numbers using Base 10 and then add these together. They move onto "exchanging" ten "ones" for a "ten".   	Children represent the Base 10 apparatus using lines and crosses. Children will understand how bar models and part-whole models can represent addition.    	Children will be able to partition numbers into tens and ones (or hundreds, tens and ones) to add together, using informal jottings to record their calculations. **132 + 225 = 357** **100 + 200 = 300** **30 + 20 = 50** **2 + 5 = 7** **300 + 50 + 7 = 357** Children will then begin to use the column method to add, initially using the expanded form of the calculation when they begin to exchange. They will understand that the equals (=) sign shows a balancing of a number sentence. T O 3 5 + 2 2 5 7 T O 3 5 + 2 7 1 2 5 0 6 2

<table>
<tr>
<td>Adding 9, 99, or other near multiples</td>
<td>Children will learn that it is more efficient to add a near multiple and adjust than adding 9, 99,199 mentally.
 
</td>
<td>Children will use a number line to show adding 100 and then subtracting 1 for adding 99, or add 10 subtract 1 for adding 9.
 

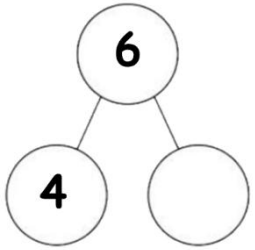
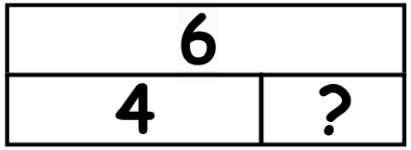
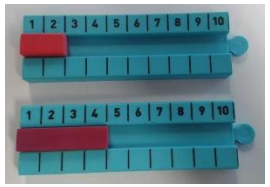
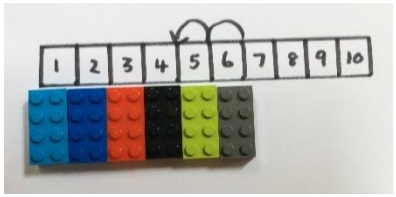
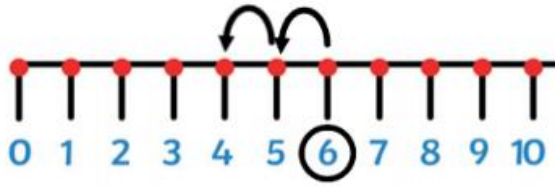

</td>
<td>Children will progress to working out this problem mentally and recording the number sentences.
 
</td>
</tr>
<tr>
<td>Adding numbers > 3d without exchanging</td>
<td>Use place value counters on a place value grid.
 
</td>
<td>Draw out the place value grid with the counters represented.
 
</td>
<td>Formal column method. Children will understand that they needs to start with the ones first. The addition symbol will be placed on the left of the calculation.

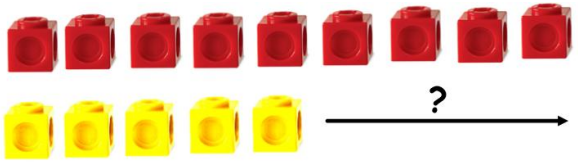
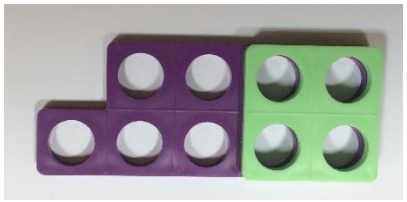
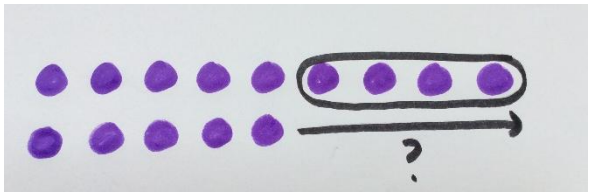
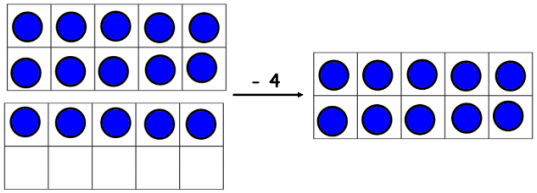
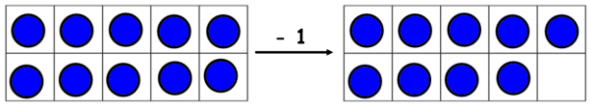
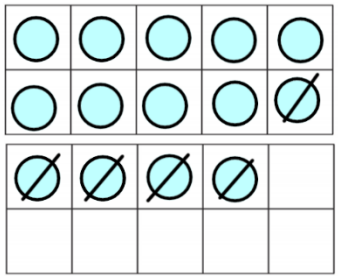
	TH	H	T	O
	2	3	2	1
+	1	1	2	1
	3	4	4	2

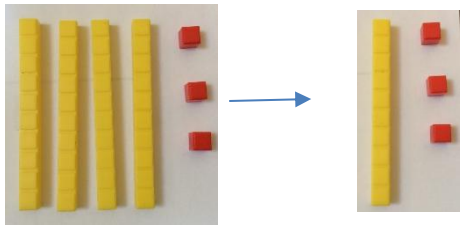
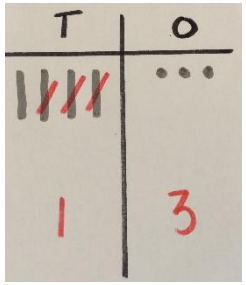
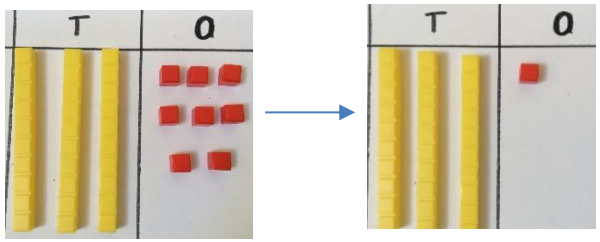
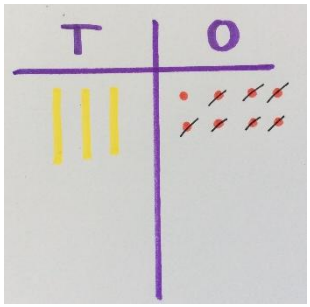
</td>
</tr>
</table>

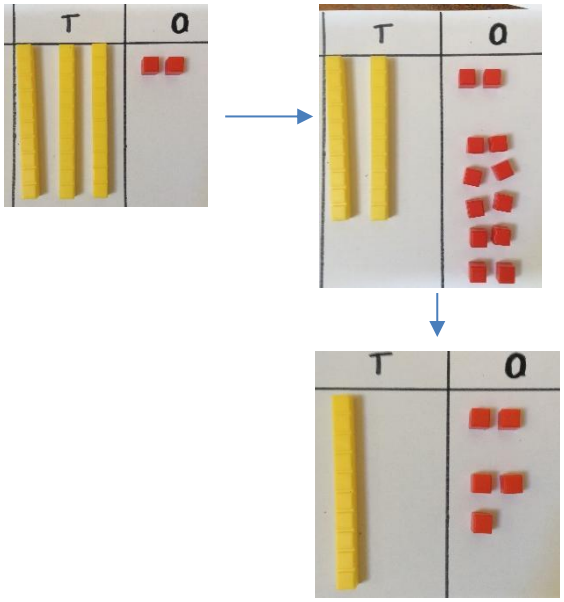
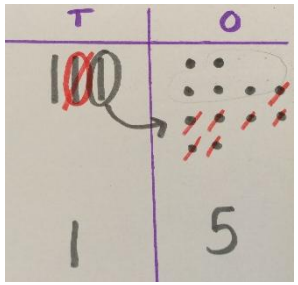
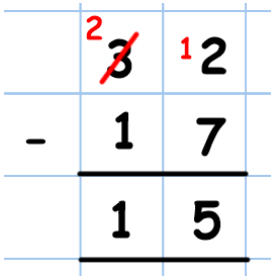
Adding numbers > 3d with exchanging	Children will physically "exchange" 10 counters for a counter in the next place value column. 	Draw out the place value grid with the counters represented and "exchanges" circled. Represent word problems as bar models. 	Formal column method with exchanges shown **underneath**. Children will be encouraged to use the language **"exchange"** and say how many exchanges will be needed. 
Adding decimals	Use decimal place value counters on a place value grid. 	Draw out the place value grid with the counters represented. Children will represent word problems and calculations as bar models.  	Formal column method. Ensure children are confident in laying out the calculation, lining up the place value columns and decimal point and using zero as a place holder for calculations which have a different number if decimal places.. Exchanges shown underneath as in column addition above. 

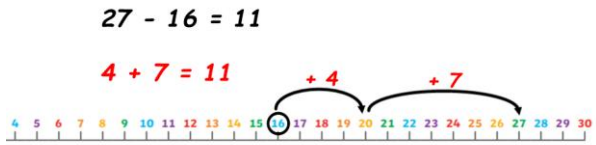
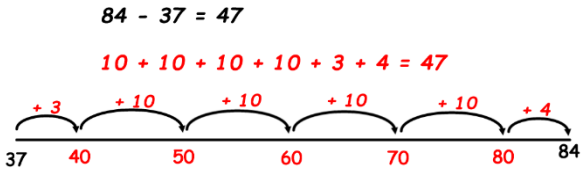
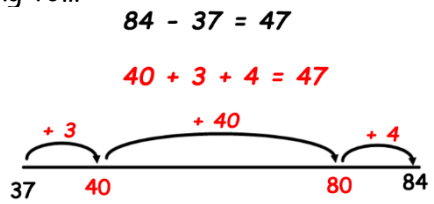
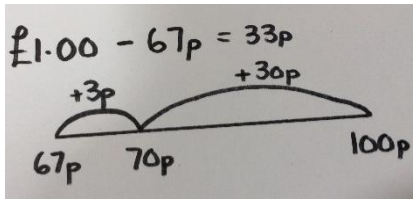
Progression in Subtraction

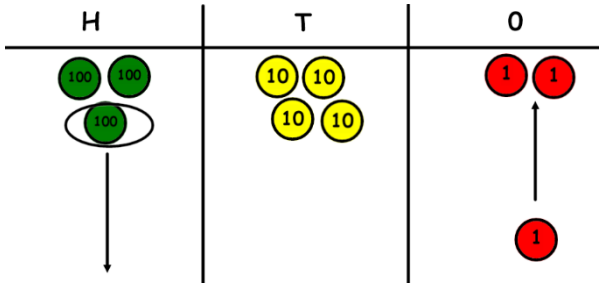
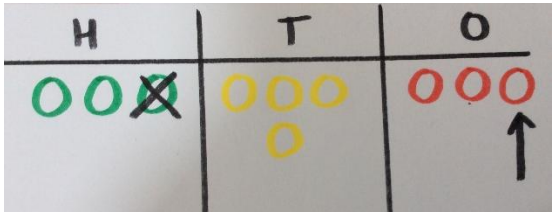
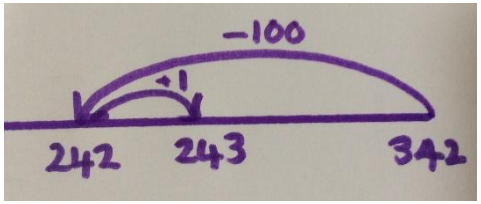
	CONCRETE	PICTORIAL	ABSTRACT
Taking away	Children use a variety of different resources in different contexts physically take away a number of objects from a set.  	Children will draw the physical resources and cross out as they subtract. 	Connect the children's understanding to the part-whole model and bar models early on.  
Counting back	Children begin to count back using objects arranged along a number line or number track.  	Children use number lines to count back and represent what they see pictorially. 	As children become more confident with counting back they should begin to develop known number facts. Children will be beginning to understand the (-) symbol and see this represented in a calculation. **6 – 2 = 4** **4 = 6 - 2**

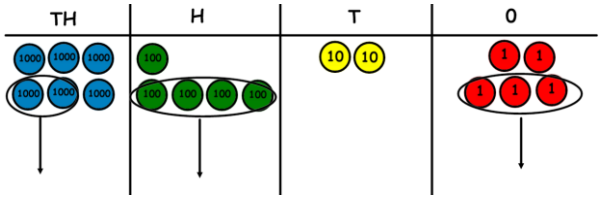
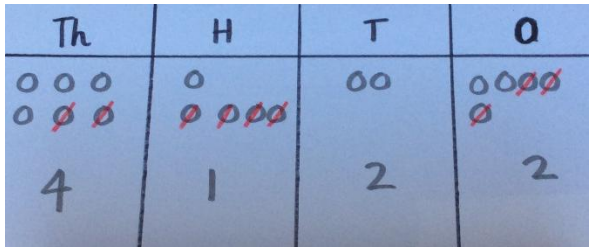
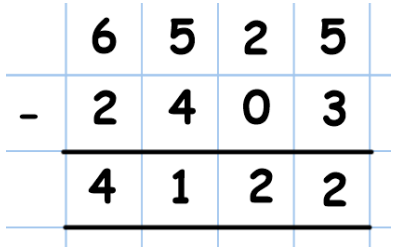
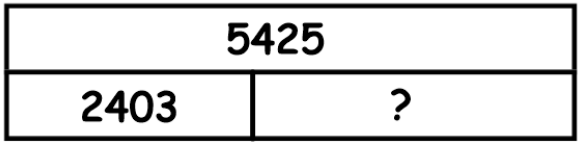
Finding the difference	Children need to encouraged to "spot the difference" before beginning this step. Children line up the two values and find the difference between the two. Numicon can also be used to help children "spot" the difference.  	Children will draw the concrete objects and use bar models to find the difference.   	Children should understand that the numbers can be given in any order and the difference is the same. The can then use a corresponding expression using the (-) symbol. **"The difference between 4 and 9 is 5"** **"The difference between 9 and 4 is 5."** **9 – 4 = 5**
Subtracting crossing the ten	Children use ten frames to make the larger number and then subtract making ten first then continuing. **14 – 5 = 9**  Then…  	Children present ten frames pictorially and cross off as they subtract. 	Children need to know that they can partition the number out to subtract, bridging through the ten. Children will need a lot of consolidation of this to enable them to build fluency. **14 – 5 =** **4 1** **14 – 4 = 10** **10 – 1 = 9**

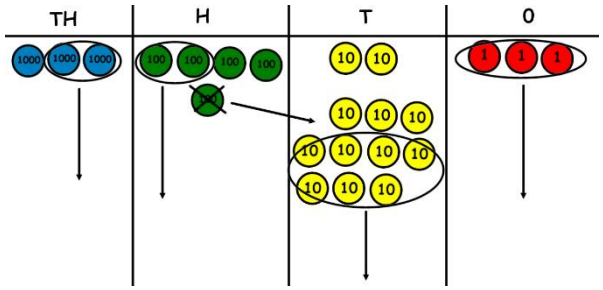
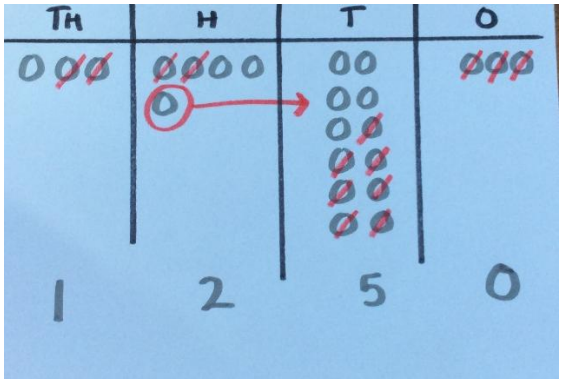
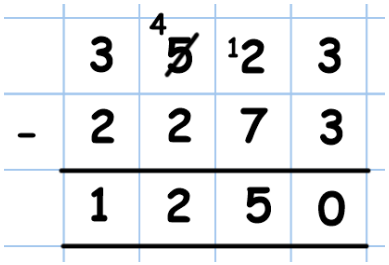
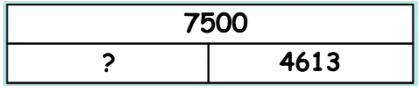
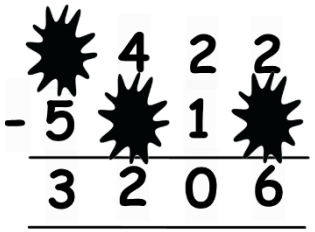
10, 100 less	Children use Base 10, progressing to place value counters as they reach KS2. Children need to discuss and spot patterns about which place value columns are affected when we subtract 10s/100s. 	Children draw out the Base 10 or counters to help them subtract 10s or 100s. **43 – 30 = 13** 	As children become more confident they will progress to solving these calculations mentally, along with missing number problems. **43 – 30 = 13** **13 = 43 – 30** **43 – ☐ = 13**
Subtracting 2d from 2d (not crossing 10)	Children use Base 10 to represent the larger number which they can physically subtract from. 	Children represent Base 10 pictorially. **38 – 7 = 31** 	Children progress from using concrete and pictorial resources to representing calculations using formal columnar methods. The subtraction symbol is written on the left of the calculation. ***It is important to encourage children to decide whether they need to write a formal method or calculate mentally.*** 3 8 – 7 3 1

Subtracting 2d from 2d (crossing 10)	Children use Base 10 to explore exchanging one ten for ten ones, then subtracting by physically removing the resources. 	Children continue to draw out the Base 10. 	Children progress to formal columnar subtraction with an **"exchange"**. They must understand that when we exchange one ten for ten ones, we still have the same amount. It is important that the children are familiar with partitioning in different ways to help them understand this. 

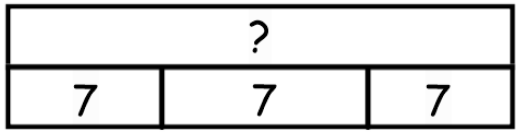
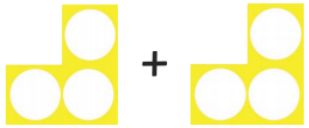
Subtracting using counting on	Children use pre-drawn number lines to help them count on from the smaller number to the larger number. They need to know their number bonds to 10 well so they can count onto the next ten.  	Children draw their own number lines to count on from the smaller number to the larger number.  Progressing to…  Or…  	Children will gradually build up the mental skills required to count on without having to draw the number line, possibly making some mental jottings. This skills is particularly effective when working out the difference between two numbers which are fairly close together or working out change. **84 – 37 = 47** Children will also learn to their knowledge of number bonds to ten and 100 to "count up" to a whole multiple of 10, 100 or 1000. Children need to be confident with number bonds to ten to access this step. A number square or number line can help with this.

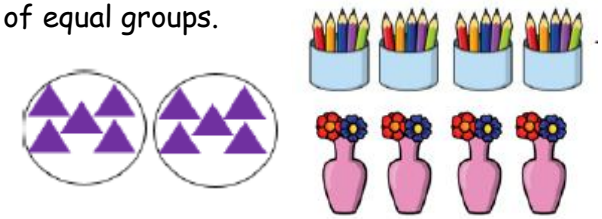
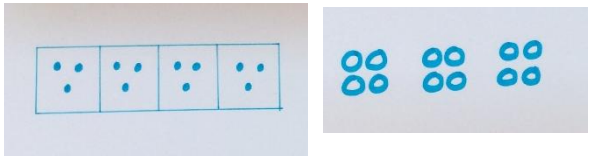
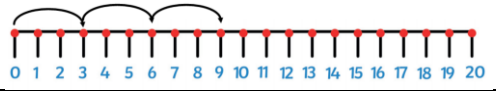
Subtraction using near multiples	Children use place value counters to help them subtract 99 by subtracting 100 and then add 1 in order to complete the calculation more efficiently. Children will need to be clear that by adding 100 they have added "more" than required and need to adjust. **342 – 99 = 243** 	Children represent pictorially.  	Children use mental skills to complete the adjustments. ***Note: Children will need to confident in calculating 100 less before attempting this skill. See previous step.*** **342 – 99 = 243** **342 – 100 = 242** **242 + 1 = 243**

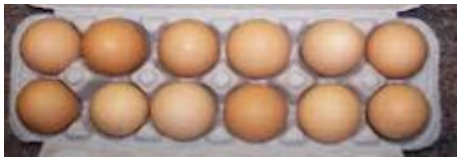
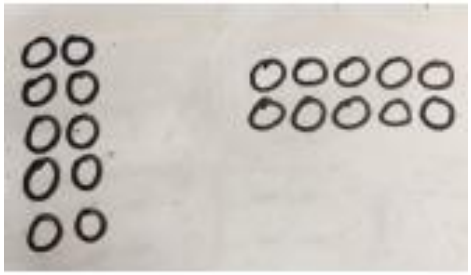
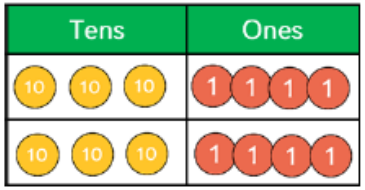
Subtracting numbers > 2d (no exchange)	Children are introduced to column subtraction using place value counters. 	Children draw out the place value counters to help them. 	Children use formal columnar subtraction, beginning with the **"ones"** digit. Children need to understand that the larger number is written first, flowed by the smaller number. ***It is important to encourage children to decide whether they need to write a formal method or calculate mentally.***  Children then begin to solve missing number problems. 

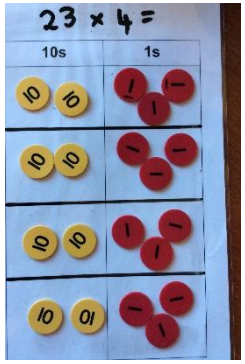
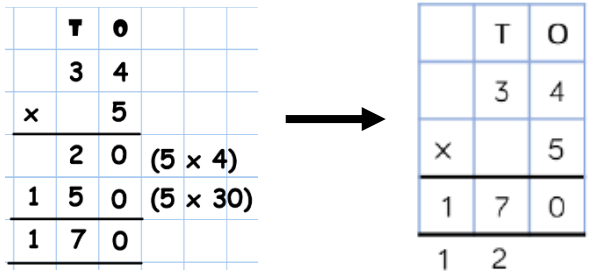
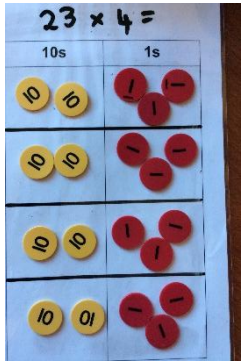
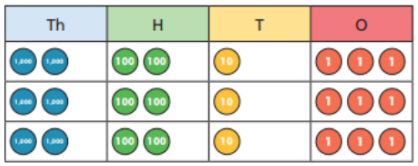
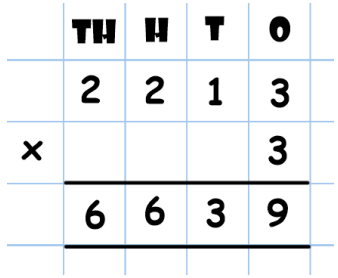
Subtracting numbers > 2d (exchange)	Children represent their calculation using place value counters and **"exchange"** a larger column for a smaller one. Children need to understand that they are doing the top number subtract the bottom number. 	Children draw out the place value counters to help them. 	Children use formal columnar subtraction. They need experience of subtracting where there are zeros in the larger number. ***It is important to encourage children to decide whether they need to write a formal method or calculate mentally.***  Children then begin to solve missing number problems.  

Progression in Multiplication

	CONCRETE	PICTORIAL	ABSTRACT
Bar models are used throughout all year groups to demonstrate multiplying:			
Doubling	Children learn that double means "twice as many". Children should be given the opportunity to make doubles with a variety of real-life objects.  	Children can represent doubles pictorially in a number of ways by drawing resources they are familiar with and part-whole models.  	Children can complete number sentences relating to doubling. **Double 4** **4 + 4 =** **2 x 4 =** ***Note*: x symbol introduced in Y2.**

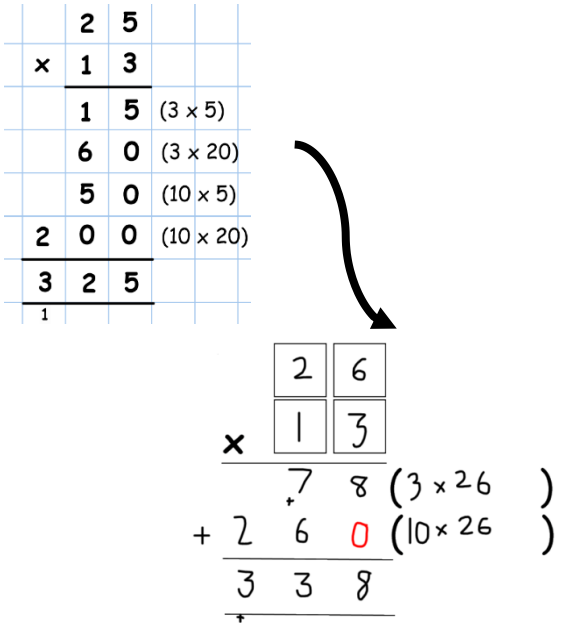
Make equal groups	Children can say whether groups are "equal" or "not equal" and begin to make equal groups using real-life objects. 	Children use pictures to find out how many equal groups and how many in each group. They also draw their own representations of equal groups.  Josh is drawing equal groups of 3  Complete his drawing.	Children can complete stem sentences describing the equal groups before moving on to understand that equal groups can be represented using the "X" symbol. **2 x 3** **There are two equal groups of 3.** **There are two 3s.**
Add equal groups	Children make equal groups of a value using concrete resources. ***Stem sentence: There are three equal groups with 3 in each group. There are 9 altogether.***  = 9	Children use bar models to show that there are equal groups and draw their own equal groups.  Children may then begin to make equal jumps on number lines, which will reinforce counting skills.  0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	Children use number sentences to represent the concrete or pictorial equal groups they have made. **3 + 3 + 3 =**

Make arrays	Children begin to make arrays by building counters or other concrete resources up in "columns" and "rows". 	Children draw arrays in the form of counters in their journals to support their thinking. 	Children can use their array to write a number of calculations. E.g.  **5 + 5 + 5 = 15** **3 x 5 = 15** **5 x 3 = 15** **15 = 5 + 5 + 5**
Multiply 2d by 1d by partitioning	Children can use Numicon, Cuisenaire rods, base 10 or place value counters to demonstrate or support their thinking.	Children can draw counters or Base 10 equipment to demonstrate their thinking. **34 x 2 =** 	Children can make informal jottings to partition, leading to increased ability to complete mental calculations. **13 x 4 = 52** **10 x 4 = 40** **3 x 4 = 12** **40 + 12 = 52**

Multiply 2d by 1d (expanded method to formal)	Children represent their calculation by using place value counters placed clearly in rows depending on the multiplier. 	Children make pictorial representations of their calculation by using place value counters placed clearly in rows depending on the multiplier. 	Children begin by consolidating partitioning by learning how to represent their calculations formally using the expanded method, then quickly transfer this knowledge to the formal method. Brackets are useful to illustrate thinking when using the expanded method. ***Note: The multiplication symbol is placed at the left of the calculation.*** 
Multiply numbers = or > 2d by 1d	Children represent their calculation by using place value counters placed clearly in rows depending on the multiplier. 	Children can continue to represent their calculation using place value counters. 	Children use the formal method. Where necessary, exchanges will be shown underneath the line. 

<table>
<tr>
<td>Multiply Numbers = or > 2d by 2d (grid)</td>
<td>Children should be familiar with multiplying numbers = or > 2-digits by a single digit number and be confident in solving these using the formal column method (abstract) before moving on to this step.</td>
<td>Children use the grid method to multiply 2d x 2d in the first instance. They explore the best way to add the digits to find their answer.

<table>
<tr><td></td><td>40</td><td>5</td></tr>
<tr><td>10</td><td>400</td><td>50</td></tr>
<tr><td>7</td><td>280</td><td>35</td></tr>
</table>
400 + 280 = 680
680 + 50 = 730
730 + 35 = 765</td>
</tr>
</table>

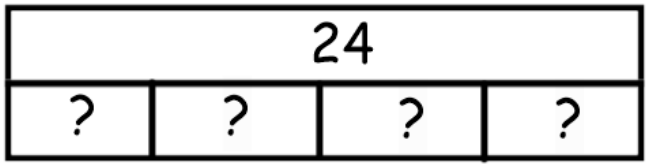
Multiply numbers = or > 2d by 2d (column)	Children should be familiar with multiplying numbers = or > 2-digits by a single digit number and be confident in solving these using the formal column method (abstract) before moving on to this step.	Children use long multiplication, first using the expanded method. They need to understand that zero is used as a placeholder when multiplying by multiples of 10 and that they are partitioning the larger number to multiply it by the smaller one. Children should be encouraged to show the brackets to demonstrate partitioning. 

Progression in Division

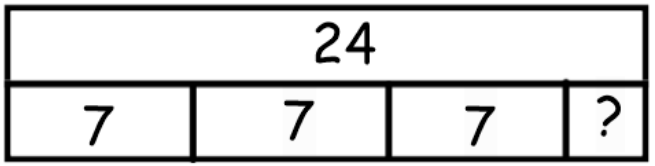
	CONCRETE	PICTORIAL	ABSTRACT
Note: It is vital that children understand that division can be expressed in two ways: **sharing and grouping**. They need to do these practically so they can apply the language of "sharing" and "grouping" to real life concepts. E.g. **Share 12 cakes between 3 people** as opposed to **there are 12 cakes and each person has 3**.			

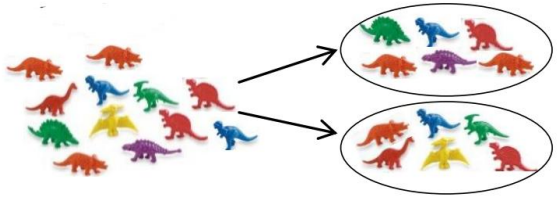
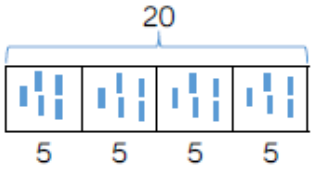
Bar models demonstrate that the whole is being split into equal parts when dividing. These are to be used throughout all year groups to ensure that they become familiar and comfortable with them.

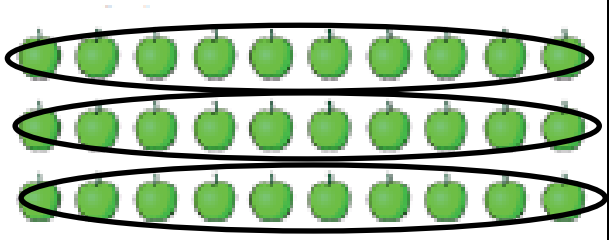
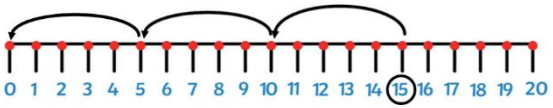
Bar model showing equal groups:

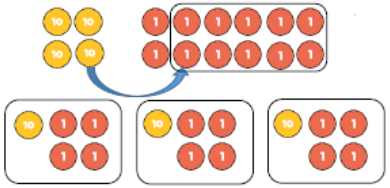
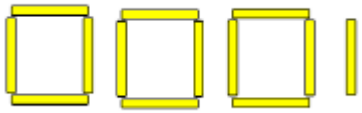
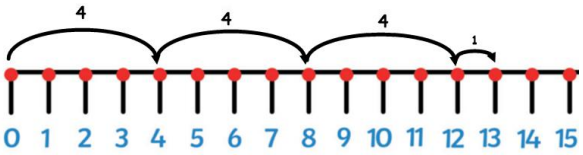


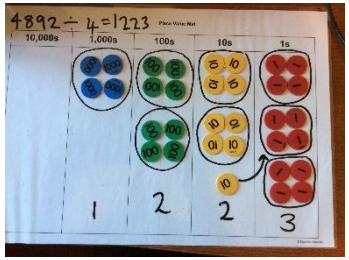
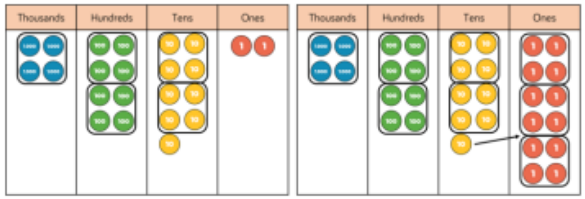
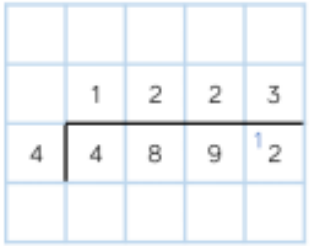
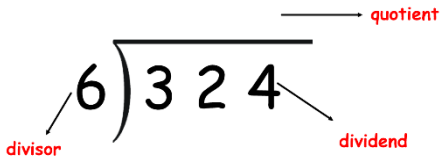
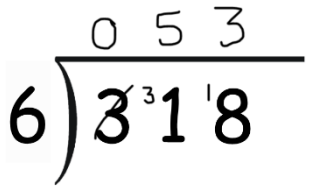
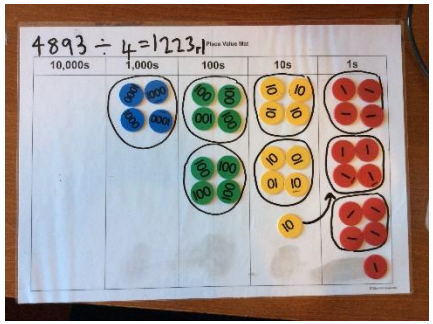
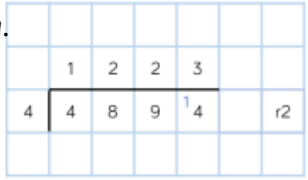
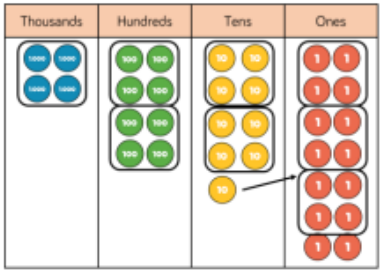
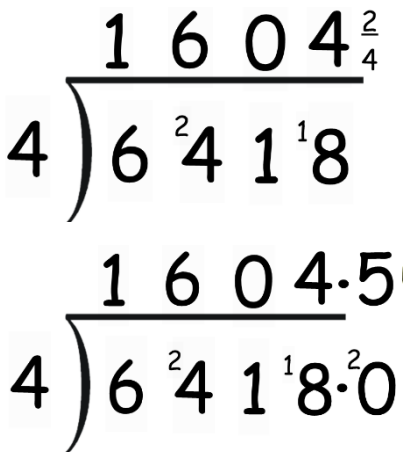
Bar model showing equal groups and a remainder:



Sharing	Children use physical resources and apparatus to share equally. 	Children represent the items being shared by drawing pictures.  8 shared by 2 is ____. Children gradually move onto sharing using a bar model: 	Children represent their calculations using the division symbol (÷). **20 ÷ 4 = 5**

Grouping	Children use physical resources and apparatus to group equally.	Children use pictorial images to demonstrate different ways that they could be grouped. **Apples are packed in bags of 10.**  Children may also use number lines to jump back in groups: There are 15 pencils. 5 pencils are put in each pot.  	Children use the division symbol to represent their calculation. **30 ÷ 10 = 3**

Divide 2d by 1d (progressing to showing remainders)	Children need practise at using their multiplication tables to help them solve these calculations by counting up and calculating what is left over. This can be done with a variety of apparatus or by using Cuisenaire rods on a number track or along a ruler. Children will then progress to trickier calculations where times tables aren't as useful. They use place value counters to solve as below. 	Children use number lines or draw physical apparatus to show their thinking.   For trickier calculations, children can draw out their place value counters.	Children are becoming more confident at using their times tables facts to calculate. They may still have access to multiplication grids to help them count up, but should be encouraged to use informal methods to support. **13 ÷ 4 = 3r1** **42 ÷ 3 = 14**

Short division	Children use place value counters to calculate division where there is no exchange in the first instance. They then quickly move onto calculations which require an exchange.  	Children draw out the place value counters alongside the short division method to bridge understanding.   	Children use the formal short division method (bus stop). They ask themselves: • How many x goes into y? • How many left over? • How many do we exchange? • What remainders are there?  
Short division with remainders	Children use place value counters as above to group values.  	Children draw out the place value counters alongside the short division method to bridge understanding.   	As children become more confident they express remainders as a fractions, then as a decimal. 

Long division	Children should be familiar and confident with dividing numbers using the formal short division (abstract) before moving on to this step.	Long division is an alternative to short division when dividing by larger numbers. Writing multiples at the side is encouraged for accuracy and efficiency. Children start with expanded form, moving onto traditional form, larger numbers and remainders.